Introduction to Folding Twist Boxes Activities

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In the accompanying activities, students fold two-dimensional crease patterns into two-dimensional or three-dimensional geometric objects. They then use geometric properties to understand how each crease pattern produces a particular geometric model. The crease patterns are based on designs by Tomoko Fuse, an expert in origami box design and one of the most published origami authors in Japan. Fuse published techniques that produce boxes shaped like prisms with regular polygon bases [2]. The instructions given here are simplified for inexperienced folders and include crease patterns for students to cut out and fold. (Traditional origami instructions begin with a blank piece of paper.)

Cognition. These discovery-based activities assist students in developing their mathematical reasoning and sense-making skills. In order to understand why and how these crease patterns form boxes with regular polygon bases, students must connect their knowledge of geometry to the creases in the patterns.

Prerequisites. Students should be familiar with complementary and straight angles, the angle sum property of triangles, properties of right, equilateral and isosceles triangles, and the Pythagorean theorem. Knowledge of regular polygons is a prerequisite for most activities, but this concept is introduced/defined in *Activity: Folding Rectangles into Regular Polygons*. The two activities *Activity: What's up with h as n gets larger and larger?* and *Activity: How much excess is there in a messy box bottom?* require familiarity with limits and could be used as a nice application of L'Hôpital's rule.

Notes on the Activities. Multiple activities are given and they do not all need to done or done in the order given. However, all explorers must start with the *Activity: Fold Four Boxes*, which consists entirely of folding two-dimensional crease patterns into three-dimensional hollow prisms that are open at one end (or more simply polygonal-based open boxes). The crease patterns in *Activity: Fold Four Boxes* are given in order of folding difficulty. Thus, we recommend starting with the square-base box and completing additional boxes as the folding abilities of your students allow. The twist fold in the center (Step 6) is challenging—more so for the pentagon- and hexagon-base boxes—and most students will need assistance, so it is well worth practicing ahead of time. The triangle-base box has some extra paper inside the box that must be folded down to avoid distorting the box. Student ability to fold precisely varies significantly, and if they work in groups, students who cannot complete the activity can at least see other students' finished boxes. We recommend having students fold at least one box to motivate the other activities.

Activity: Folding Rectangles into Regular Polygons and *Activity: Find Crease Patterns for n-gon-base Twist Boxes* both ask students to explore why the crease pattern produces a box with a regular polygon base and to generalize this crease pattern to create a polygon-base box with n sides. *Folding Rectangles into Regular Polygons* is a simplified version of the other activity both in terms of folding (it only examines portions of the crease pattern that fold to two-dimensional objects, which are much easier to fold than the three-dimensional objects) and mathematically (students are guided more closely to examine the different parts of the crease pattern that create the interesting geometric structure). We do not recommend doing both of these activities, but picking the one that seems most appropriate for the level of your students.

Activity: Analyze "Clean" vs. "Messy" Box-Bottom Twists explores the difference in symmetry between polygon-base boxes with even or odd numbers of sides. It could be completed directly

after *Activity: Fold Four Boxes* (and without either of the activities about understanding the crease pattern).

Activity: What's Up With h as n Gets Larger and Larger? could also be done directly after *Activity: Fold Four Boxes* but works best after students have already discovered the relationship between the angle in the crease pattern and the interior angle of a regular polygon in either activity *Activity: Folding Rectangles into Regular Polygons* or *Activity: Find Crease Patterns for n-gonal-Base Twist Boxes.*

Activity: How Much Excess is *There in a Messy Box Bottom?* requires that students have at least folded the four boxes and completed *Activity: Analyze "Clean" vs. "Messy" Box-Bottom Twists.*

References

[1] S. Fujimoto and M. Nishiwaki, Sojo Suru Origami Asobi Eno Shotai (Invitation to Creative Origami Playing), *Asahi Culture Center*, 1982.

[2] Fuse, Tomoko. *Boxes in One Piece.* Chikuma Shobo, Tokyo, 1992. (in Japanese; ISBN: 4480872035)

[3] Fuse, Tomoko. *Origami Gift Boxes.* Chikuma Shobo, Tokyo, 2000. (in Japanese; ISBN: 4480873236)